mann constant; T, T(r), T_0, T_1, temperatures; I, light flux intensity; u, u(r), displacements of points of the medium; α, volume expansion coefficient; σ, Poisson's ratio; E_1, Young's modulus.

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NUMERICAL INVESTIGATION OF PHOTOABSORPTION CONVECTION IN A HORIZONTAL TUBE.

II. UNSTEADY-STATE CONVECTION

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The establishment of photoabsorption convection in liquids, gases, and plasma subjected to horizontal laser beams of different intensities is investigated by numerical integration of the Navier–Stokes equations in the Boussinesq approximation.

The first part of this investigation [1] was devoted to numerical modeling of steady-state photoabsorption convection. In correspondence with the results of dimensional analysis [2] we obtained three steady-state convection regimes differing in the index of the power in the exponential relation between the free-convection velocity and the laser heating intensity. In the second part of the work we first carried out a numerical investigation of the establishment of the steady state in time. The calculations were conducted by the same method [3] as in Part I.

The establishment times are in good agreement with the estimates in [2]. The processes of establishment of the different regimes differ in their nature: In the case of weak convection all the parameters vary monotonically in time, whereas in the case of developed convection there are considerable oscillations which rapidly decay with time. The effect of modulation of the laser radiation on the establishment process is also considered.

Statement of the Problem. We solve the problem of convective motion caused by absorption, in a long cuvette of rectangular cross section, of laser radiation propagating parallel to the cuvette axis (Fig. 1), which is given by the Navier–Stokes equations in the Boussinesq approximation:

$$\frac{\partial\omega}{\partial t}+\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial y}\,\omega\right)-\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial x}\,\omega\right)=\Delta\omega+\frac{\partial T}{\partial x},$$
$$\frac{\partial T}{\partial t}+\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial y}T\right)-\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial x}T\right)=\frac{1}{\mathrm{Pr}}\,\Delta T+qf(x,\ y), \tag{1}$$
$$\Delta\psi=-\omega,\quad u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}.$$

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The dimensionless quantities in (1) and henceforth are denoted by the same symbols as the corresponding dimensional quantities; in places where this might lead to confusion the dimensionless quantities are denoted by a tilde. As quantities with independent dimensions we take L (the width of the cavity), ν, and β.

On the cuvette walls we assign the boundary no-slip and no-flow conditions

$$\psi = 0, \quad \frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial y} = 0;$$

the wall temperature is kept constant at T = 0. The profile of the laser beam is assumed to be Gaussian

$$f(x,\ y) = \exp\left(-\frac{(x - L/2)^2 + (y - H/2)^2}{a^2}\right).$$

Establishment of Convective Motion Due to Laser Radiation. In [2] dimensional analysis of the Navier–Stokes equations was used to estimate the times t_T and t_V of establishment of the temperature and velocity of photoabsorption convection. Depending on the dimensionless heat release q we obtained the following expressions for the characteristic times of establishment of convection within a laser beam of radius a:

for weak convection

$$t_T = a^2/\chi, \quad t_V = \max(a^2/\nu,\ t_T); \tag{2}$$

for moderate convection

$$\begin{aligned} t_T &= a/V, \quad t_V = \max(a^2/\nu, \quad t_T), \quad \Pr \gg 1, \\ t_T &= a^2/\chi, \quad t_V = \max(a/V, \quad t_T), \quad \Pr \ll 1, \end{aligned} \tag{3}$$

where V is the characteristic velocity of convection;

for developed convection

$$t_T = t_V = a/V.$$

In the case of moderate convection we have the following inequalities: When $\Pr \gg 1$ $\chi/a < V < \nu/a$, whence $a^2/\chi > t_T > a^2/\nu = t_V$; when $\Pr \ll 1$ $\nu/a < V < \chi/a$, whence $a^2/\nu > t_V > a^2/\chi = t_T$.

When a is replaced by L/2; we obtain the characteristic establishment times for the whole volume of gas or liquid.

To determine the establishment times we obtained plots of the temperature and velocity at two typical points in the cavity against time (Fig. 2). As such points we took the center of the laser beam and point A (Fig. 1) on the periphery, whose selected position satisfied the following conditions: It had to be as far as possible from the laser beam; it had to be some distance from the cuvette wall; the path of the liquid particle from the beam center to point A had to be such that neither diffusion nor convective transfer had unilateral advantages.

When these conditions are fulfilled we can assume that the establishment of the parameters at point A characterizes establishment in the whole cuvette. Various criteria can be used for determination of the establishment time.

In Table 1 the establishment time is the time in which the parameter attains a value equal to half of the steady-state value. For comparison the figures in the brackets in Table 1 give the establishment times calculated from the estimates in [2]. It is apparent that there is good general qualitative agreement. Dimensional analysis allows highly accurate evaluation of the order of the quantities. It should be noted specially that the establishment times for the velocity in the zone of the laser beam and throughout the cavity were approximately the same and depended on the beam diameter. The reason for this is that the estimates of [2] must be applied with caution to closed volumes, since an infinite volume, not bounded by walls, was considered in [2]. In a closed volume the motion of the liquid in one part is transferred at the velocity of sound to other parts owing to the continuity of the flow. This fact is clearly revealed by the different behavior of the curves $T^A(t)$ and $V^A(t)$ at the initial times: The temperature at point A begins to increase after a delay, whereas the velocity increases from the very beginning.

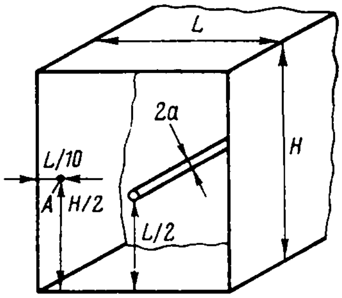

Fig. 1. Diagram showing position of beam in tube.

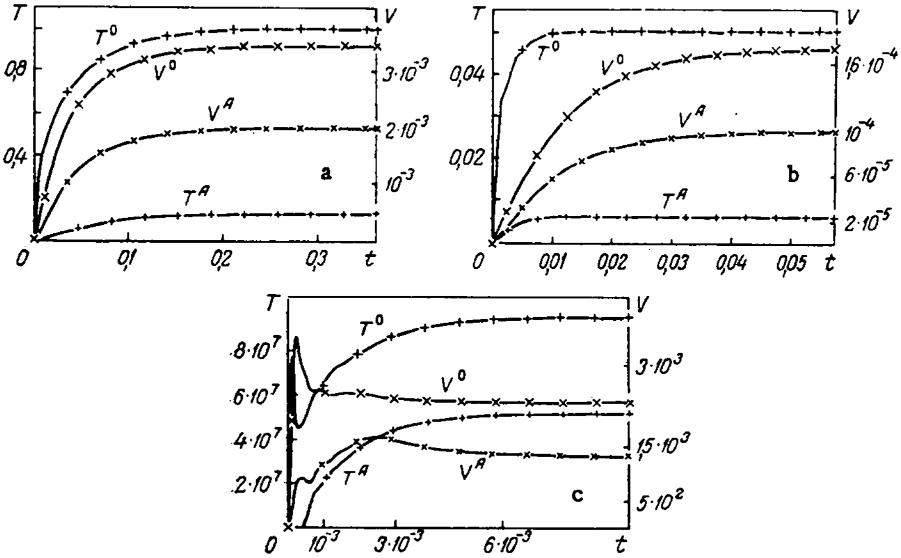

Fig. 2. Establishment of temperature and velocity at center of beam and point A at wall: a) q = 100; Pr = 1; b) 100 and 0.05; c) 10^{12} and 1. L = H, $a/L = 0.2$.

The second difference is the violation of the selection rules formulated in [2], according to which the establishment time of the velocity field associated with convection cannot be less than the establishment time of the temperature field, i.e., we must have $t_V \geqslant t_T$. This difference is due entirely to the selected method of determining the establishment time from the graph, which does not give the establishment time of the steady state, but is a characteristic of the rate of increase of the physical parameter and can be used even in the case where there is no steady state, e.g., if the established regime is oscillatory. If by establishment time we mean the time in which the relative deviation of the parameter from its steady-state value becomes less than some small prescribed value, the selection rule $t_T \geqslant t_T$ will be valid. The time dependences shown in Fig. 2 correspond to steady-state operation [1] and are plotted in dimensionless variables.

The establishment pattern, like the structure of convection, is determined by the interval in which the value of the heat release parameter q lies. When q is small all the quantities increase monotonically with time and gradually level out. The curves have the same shape (Fig. 2a), except in the case of plasma Pr = 0.05 (Fig. 2b), where, as dimensional analysis shows [2], the establishment of the temperature field occurs much earlier than the establishment of the velocity field. In the case of moderate (even if the regime with $V \sim q^{1/2}$ is absent) and developed convection establishment ceases to be monotonic and oscillations arise (Fig. 2c). The reason for the oscillations is that in this range of q the heat from the beam zone is carried away by the flow of liquid, which possesses inertia, and, hence, there is a time discrepancy between the temperature and velocity. While the velocity of the liquid is low, the convective flow does not manage to remove the heat, and the temperature at the center increases to values greater than the steady-state value. As a result the repul-

TABLE 1. Dimensionless Times ($\tilde{t} = t\nu/L^2$) of Establishment of Temperature T and Velocity V at Center of Laser Beam (I) and at Point A (II) (L = H, $a/L = 0.2$)

q		10^2		10^7		10^{12}	
Pr	I, II	T	V	T	V	T	V
1	I	0,01 (0,01)	0,02 (0,01)	0,001	0,005	10^{-4} ($3\cdot10^{-5}$)	10^{-4} ($3\cdot10^{-5}$)
	II	0,05 (0,25)	0,04 (0,25)	0,03	0,007	10^{-3} (10^{-4})	$2\cdot10^{-4}$ (10^{-4})
20	I	0,3 (0,2)	0,3 (0,2)	0,01 (0,003)	0,005 (0,01)	10^{-4} ($3\cdot10^{-5}$)	10^{-4} ($3\cdot10^{-5}$)
	II	1 (5)	0,5 (5)	0,1 (0,01)	0,005 (0,25)	10^{-3} (10^{-4})	$2\cdot10^{-4}$ (10^{-4})
0,05	I	0,001 (0,0005)	0,01 (0,01)	0,001 (0,0005)	0,01 (0,01)	10^{-4} ($3\cdot10^{-5}$)	10^{-4} ($3\cdot10^{-5}$)
	II	0,003 (0,01)	0,01 (0,25)	0,003 (0,01)	0,01 (0,05)	10^{-3} (10^{-4})	$2\cdot10^{-4}$ (10^{-4})

Note. The values obtained from the estimates in [2] are given in brackets.

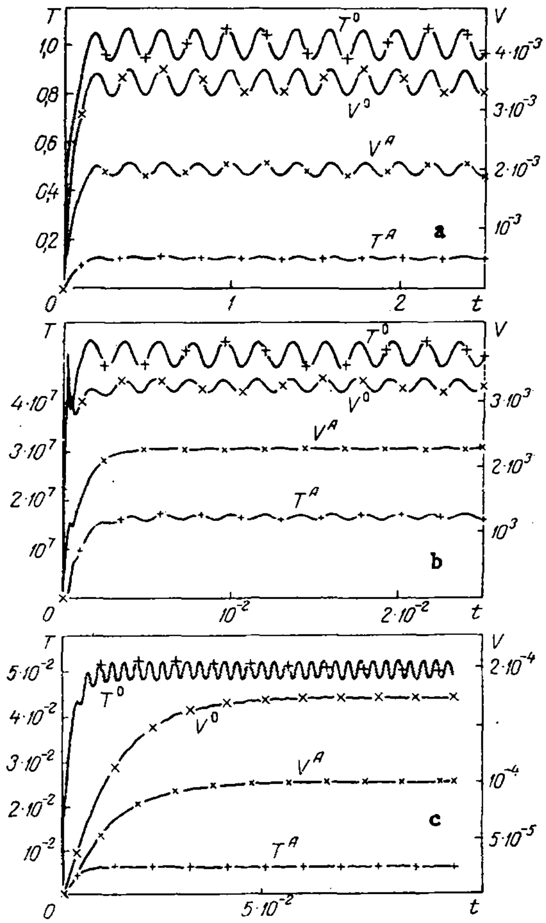

Fig. 3. Effect of time modulation of laser radiation intensity on establishment of convection: a) q = 100, Pr = 1, Ω = 10π, δ = 0.1; b) 100 and 0.05, 600π and 0.1. L = H, $a/L = 0.2$.

sive force exceeds the equilibrium value, and the liquid is accelerated to velocities greater than in the steady state, which leads to a reduction of temperature and, hence, repulsive force, and self-oscillations arise. In a viscous fluid (Pr = 20) the oscillations have a much shorter period, since the horizontal temperature gradients are particularly high owing to the low heat conduction. In the developed convection regime these oscillations are even more pronounced. The oscillations of the values at point A are less pronounced, especially in the case of temperature. The more pronounced oscillations of velocity are due to the presence of the solid walls. In an infinite medium they would probably be smoothed out.

Effect of Modulation of Laser Radiation Intensity on Establishment of Convection. A dimensional analysis of Eqs. (1) indicates that modulation of the laser radiation intensity

$$I(t) = I_0 f(x, y)(1 - \delta \sin \Omega t) \tag{4}$$

with frequency $\Omega \gg 1/t_T$ has no effect on the establishment process, and when $\Omega \ll 1/t_T$ the intensity of convective motion varies with the same frequency [2].

Figure 3a, b shows graphs of variation with time of the temperature and velocity at the center of the cuvette and at point A (see Fig. 1) for $q = 100$ ($Pr = 1$) and $q = 10^{12}$ ($Pr = 0.05$) obtained with $\delta = 0.1$ and $\Omega = 10\pi$, 1000π. In complete correspondence with dimensional analysis the unsteady state in numerical calculations is established when $\Omega \ll 1/t_T$ (Table 1). The mean value agrees exactly with the steady state ($\delta = 0$, $\Omega = 0$) (see Figs. 2a and 3a).

The nature of the established oscillations (Fig. 3a and b) is the same, but the initial phase of establishment retains the special features of its regime: a monotonic increase when $q = 100$ and oscillations when $q = 10^{12}$. When $\Omega \gg 1/t_T$ the establishment of a steady state coinciding with the solution for $\delta = 0$, $\Omega = 0$ is obtained.

In a plasma $t_V \gg t_T$, with the result that at modulation frequencies $1/t_T > \Omega > 1/t_V$ dimensional analysis [2] predicts a regime in which there are temperature oscillations but no velocity oscillations. This regime is obtained in numerical calculations (Fig. 3c) when $\Omega = 600\pi$.

From the results of the numerical experiments reported above we can conclude that dimensional analysis of the Navier–Stokes equations allows a fairly accurate prediction of several important features of steady-state photoabsorption convection and of the nature of establishment of the steady state.

NOTATION

$q = \alpha I_0 L^5 \beta g/\rho c_p \nu^3$, dimensionless heat release due to absorption of laser radiation; α, absorption coefficient; I_0, intensity on laser beam axis; L, H, width and height of cuvette; T_0, initial temperature of medium; $\chi = k/\rho c_p$, thermal diffusivity; k, thermal conductivity; ν, kinematic viscosity; c_p, specific heat at constant pressure; β, thermal expansion coefficient; g, gravitational acceleration; $\mathbf{V} = (u, v)$, velocity vector; a, radius of laser beam; ψ, stream function; ω, vorticity; Pr, Prandtl number; T^0, V^0, temperature and velocity on beam axis; T^A, V^A, temperature and velocity at point A; t_T, t_V, establishment times of temperature and velocity; Ω, frequency of modulation of laser beam intensity; δ, modulation amplitude.

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